

# How to focus a Cherenkov telescope

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**Abstract.** Cherenkov telescopes image the Cherenkov emission from air showers. A priori, it is not obvious if the ‘best’ images are achieved by measuring Cherenkov photon angles, i.e. focusing the telescope at infinity, or by considering the air shower as an object to be imaged, in which case one might focus the telescope on the central region of the shower. The issue is addressed using shower simulations.

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## 1. Introduction

Imaging Atmospheric Cherenkov Telescopes (IACTs) have emerged as the most successful instrument for gamma-ray astrophysics in the TeV energy range (e.g., Weekes 1996). IACTs image the Cherenkov light emitted by an air shower onto a highly sensitive photon detector (the ‘camera’), typically a matrix of photomultiplier tubes (PMTs). Viewed with a telescope located at a distance of  $O(100 \text{ m})$  from the shower axis, the air shower generates an elongated image. The orientation of the image reflects the orientation of the shower axis, the intensity of the image the shower energy, and the width of the image relates to the shower type and can be used to distinguish gamma-induced electromagnetic showers and hadronic background showers. Combining the multiple views provided by stereoscopic systems of Cherenkov telescopes, the shower axis can be unambiguously located in space; typical event-by-event precision is  $0.1^\circ$  for the direction of the shower axis, and  $\approx 10 \text{ m}$  for the location of the shower impact point (Aharonian et al. 1997, Konopelko et al. 1999).

The optical element of IACTs is a large reflector, usually composed of a number of smaller mirror segments. With a shower light yield of about  $100 \text{ photons m}^{-2}\text{TeV}^{-1}$  and a typical photodetector efficiency around 10%, mirror areas of about  $10 \text{ m}^2$  and  $100 \text{ m}^2$ , respectively, are required to provide reasonable images ( $O(100)$  detected photons) of 1 TeV and 100 GeV gamma-ray showers. With such large mirrors, the shower image has a limited depth of field, and one needs to decide how to focus the telescope:

- One option is to image photon directions, i.e. focus the telescope at infinity by locating the camera at a distance  $f$  from the mirror, where  $f$  is the focal length. In the small-angle approximation, the (‘plate’) coordinate  $\vec{q}$  of a photon detected in the camera plane measures the slopes  $\vec{\theta} = (\theta_x, \theta_y)$  of the photon direction relative to the telescope axis (which defines the  $z$  direction)  $\ddagger$

$$\vec{q} = f\vec{\theta}$$

In the following, plate coordinates  $\vec{p} = (x, y)$  will be expressed in units of degrees, dividing out the factor  $f$ :

$$\vec{p} = \frac{180^\circ}{\pi} \vec{\theta} = \frac{180^\circ}{\pi} \frac{\vec{q}}{f} \quad .$$

Since the direction of Cherenkov photons - rather than their location - relates to the direction of the primary, this may seem a natural choice.

- Alternatively, one might consider the IACT taking a photograph of the air shower, in which case one will focus the telescope at the typical distance  $S$  between the air shower and the telescope. This is achieved by locating the camera at a distance

$\ddagger$  In the convention used here, the mirror is treated like a lens, with the ‘object’ at  $-z$  and the image at  $+z$ .

$f/(1 - f/S)$  from the mirror. After appropriate rescaling of plate coordinates to achieve the same magnification, one finds

$$\vec{p} = \frac{180^\circ}{\pi} (\vec{\theta} - \vec{r}/S)$$

where  $\vec{r}$  is the point where the photon hits the mirror, relative to the center of the mirror.

A priori, it is not clear if this choice matters, and if yes, which choice is best. This note aims to settle the issue, based both on simple analytical arguments and on simulations.

## 2. Width of shower images

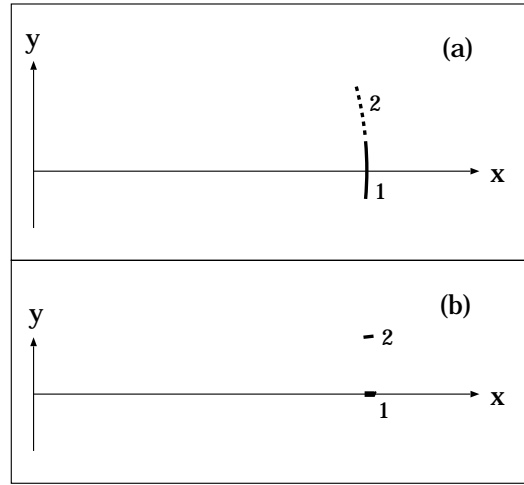
I will argue that the most important criterion is the observed width of the shower image, as compared to an ideal optical system and detector, i.e. a very small mirror with very large depth of field and a camera with very fine pixels. The width of the image is important both for the separation between narrow gamma-induced showers and wide nuclear showers, and for the angular resolution of the instrument. Any broadening of the image will worsen the cosmic-ray rejection capability of the instrument. Wider images will also make it more difficult to accurately determine the image axis, and hence the direction of the primary. In optimizing the optics, one can therefore simply aim at generating the narrowest images for gamma-ray primaries, provided that there are no other side effects. Since the reconstructed image represents a convolution of the ideal image with the instrumental response, instrumental effects can only reduce the separation capabilities between gamma-ray showers and nucleon-induced shower, and worsen the angular resolution §.

A number of effects will potentially contribute to the reconstructed width of the shower image in the camera:

- The transverse radius of the shower  $R_{shower}$  at the typical distance  $D$  from the telescope translates into a width  $\Delta\theta \approx 2R_{shower}/D$ . This dependence is used to distinguish gamma-induced showers with a characteristic value  $R_{shower} \approx 20$  m and nucleon-induced showers with  $R_{shower} \approx 70$  m (Hillas 1996). With  $D \approx 10$  km one finds a width of  $0.2^\circ$  for gamma showers ||; the core region with the high density of Cherenkov photons is quite a bit narrower.
- The finite size  $R_{mirror}$  of the mirror implies that the slopes of photons emitted from an assumed point source at distance  $D$  and collected on the mirror vary by  $\Delta\theta \approx 2R_{mirror}/D$ , when the telescope is focused at infinity. A 10 m mirror will

§ Strictly speaking, this holds under the assumption that the data analysis makes optimal use of the recorded information.

|| For these rough estimates, we use  $2R_{shower}$  as characteristic maximal distance between the emission points of two photons, and ignore the exact numerical factors arising from an integration over a circular emission area, or the actual radial distribution of shower particles. Equivalent simplifications are used in the following estimates.



**Figure 1.** Image generated by a short track segment of a particle at roughly 10 km height, viewed at a distance of about 120 m from the shower axis with a telescope with 20 m mirror diameter, in case the particle moves along the shower axis (1) or is displaced sideways by 20 m (2). The  $x$  axis is defined as pointing from the shower axis to the telescope location. (a) For a telescope focused at infinity, and (b) for a telescope focused at 10 km height.

generate a segment of  $\approx 0.1^\circ$  transverse width (Fig. 2(a)). Focused at  $D$ , the mirror will generate a point image, modulo the effects of optical aberrations discussed below (Fig. 2(b)).

- The size  $\delta$  of the camera image elements introduces an uncertainty  $\Delta\theta \approx \delta/f$  in the direction of detected photons. Minimal pixel sizes used in today's cameras are  $0.1^\circ$ ; next-generation photon detectors may allow a significantly finer segmentation of the cameras.
- Optical imperfections of the mirror, both due to imperfections in the manufacturing of the mirrors, and due to unavoidable optical aberrations in particular for off-axis rays will smear the image. The aberrations can be reduced by increasing the  $f/d$  ratio of the mirror; for Davies-Cotton mirror optics, the dominant aberration is of the scale  $\Delta\theta \approx 0.1^\circ (f/d)^{-2} \theta$  (fwhm), where  $\theta$  is the angle of rays relative to the optical axis, in units of degrees.

One notes that for a large mirror ( $R_{\text{mirror}} = 10$  m) and a modern camera (with  $0.1^\circ$  pixels) the effects of shower size, mirror size, camera pixel size, and optical aberrations for  $f/d \approx 1$  are of similar order of magnitude. Optimal focusing of the mirror then becomes a relevant issue. Image width will be minimized by focusing at the height of those shower particles most relevant for the reconstruction of shower characteristics. Focusing at the typical shower height rather than at infinity will also slightly modify the longitudinal profile of the shower, which is however much less sensitive to imaging properties since its scale is dominated by the longitudinal evolution of the shower.

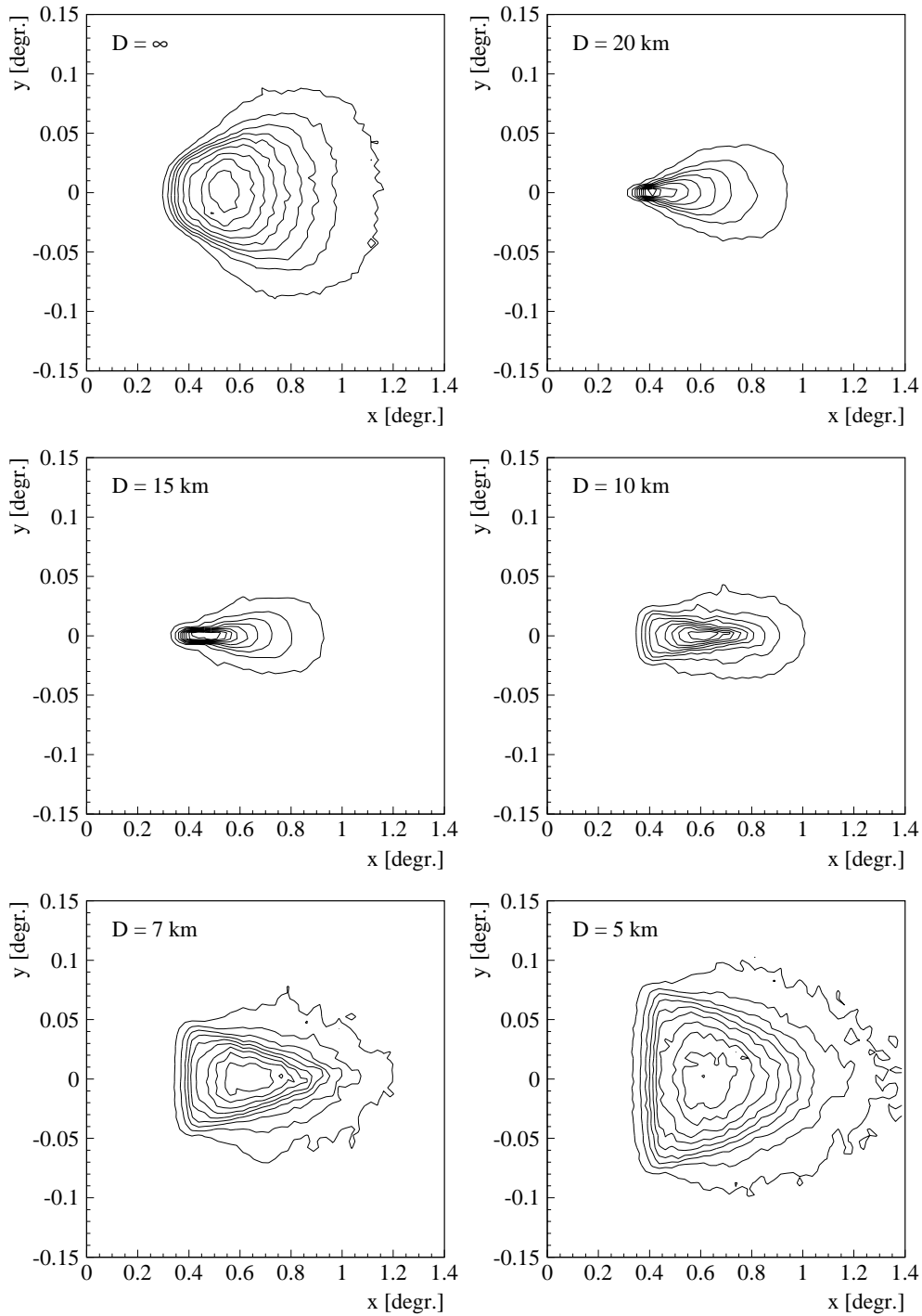
### 3. Shower simulations

To illustrate the effects on the focus distance in more detail, the distribution of Cherenkov photons in the image plane was studied in simulations using the CORSIKA air shower code (Heck et al., 1998), with modifications by Bernlöhner (2000) to include Cherenkov emission. The simulations were carried out for 100 GeV photon showers at vertical incidence, with the observation plane located at 2.2 km asl. (the height of the HEGRA installation at La Palma). A dish diameter of 20 m was assumed. In the simulation code, a reduced step size for the simulation of multiple scattering was used, to ensure proper scattering angles between the emission of successive photons hitting the mirror; with typical default values, multiple photons emitted from one straight track segment may be detected and give the (false) impression of nice Cherenkov rings. The simulations neglect optical aberrations, i.e., assume a large  $(f/d)$ . In the initial simulations, the geomagnetic field was turned off; geomagnetic fields cause additional distortions and smearing of the images, as discussed by Chadwick et al. (1999), and addressed below.

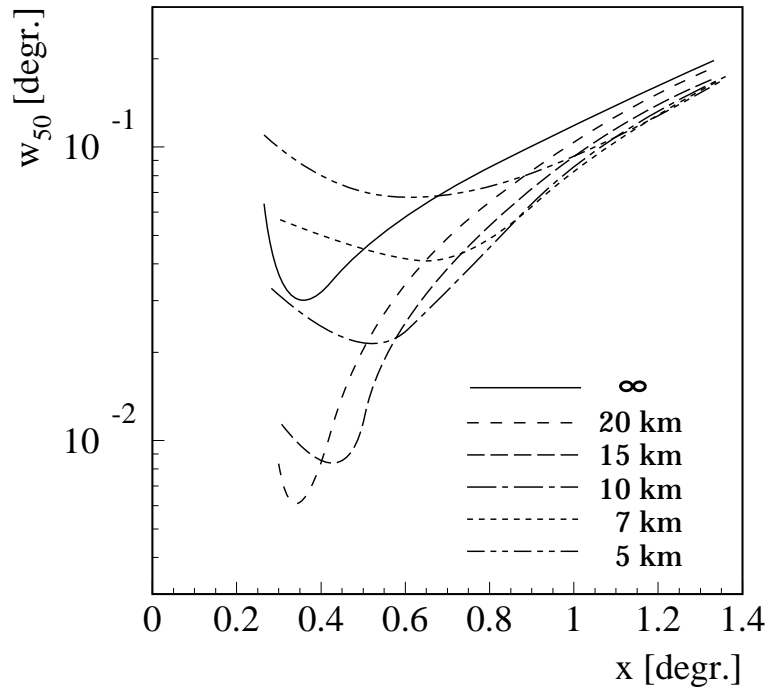
Fig. 2 shows contour lines of the average photon density in images, viewing the shower at a distance of  $R = 120$  m from the axis. The contours are given for different focus distances, varying between infinity and 5 km. In the Figure, the  $x$  axis corresponds to the major axis of the images, i.e., to the direction towards the shower axis. The  $x$  coordinate relates to the height  $h$  of the photon emission;  $x \approx (180^\circ/\pi)(R/h)$ . The  $y$  axis denotes the transverse coordinate, in which the width of a shower is determined. The influence of the focus distance is quite dramatic; a focus at 15 km to 20 km significantly narrows the small- $x$ , large- $h$  part of the image, whereas a shorter focus distance concentrates the tail end of the image. The pattern is qualitatively the same for different values of the distance between shower axis and telescope.

The contour lines in Fig. 2 tend to emphasize the sharp cusp in the center of the image, and the long tails of the distributions are not well visible; to provide a more quantitative measure, the width of images at a given  $x$  value was defined as the full width of the region in  $y$ , which – centered at  $y = 0$  – contains 50% of all photons at that  $x$  value. The result, see Fig. 3, confirms the conclusions drawn from Fig. 2. Proper focusing can reduce the width of the image by almost a factor 10 in certain  $x$ -regions, and by about a factor 2 when averaged over  $x$ . The simulations show that the longitudinal profile is virtually unaffected by the choice of the focal distance.

The optimal focus will depend on the other characteristics of the instrument and on the analysis procedures. For an instrument with small optical aberrations, i.e. sufficiently large  $(f/d)$  and with very fine pixels, and for high-intensity images one could use the low- $x$  core region of the images to obtain a very precise shower direction – a resolution in the  $0.01^\circ$  range seems feasible. For such an application, one will use a focus at about 15 km. In telescopes with larger aberrations and larger pixels, one should instead provide the optimum focus for the bulk of the image, with a focal distance around 10 km. Taking into account that most observations will be performed



**Figure 2.** Equidistant contour lines of the photon density in Cherenkov images of 100 GeV vertical gamma-ray showers, viewed at a distance of 120 m from the shower axis with a 20 m diameter reflector, focused at different distances (infinity, 20 km, 15 km, 10 km, 7 km, 5 km). The observation plane is located 2.2 km asl. Optical aberrations are not included in the simulation. Shower simulations do not include the effects of geomagnetic fields. The  $x$  axis is defined as the direction towards to telescope, i.e., the major axis of images. Note the difference in scales for the  $x$  and  $y$  axes. The spacing of contour levels scales with the peak intensity, and varies between figures.



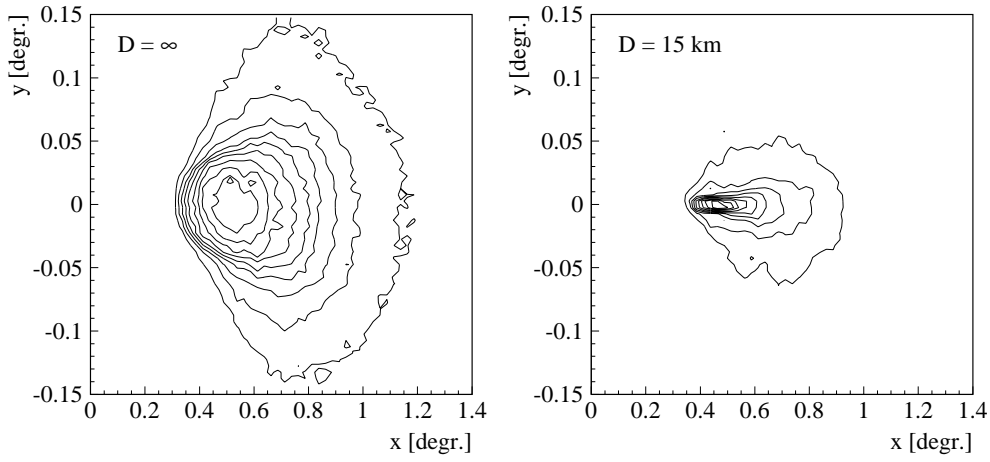
**Figure 3.** Transverse width of shower images as a function of the  $x$  coordinate along the major axis of images, for a reflector focused at distances of infinity, 20 km, 15 km, 10 km, 7 km, and 5 km. The width is defined as the full width of the range in  $y$  which contains 50% of all photons, for a given slice of the image.

at non-zero zenith angles, one might prefer a slightly larger focal distance than the two values given for vertical showers.

Depending on the location and pointing of the telescopes, the deflection of shower electrons by the geomagnetic field may be significant, and result in a distortion and smearing of the images. The influence of the geomagnetic field was studied using the La Palma field values; characteristic image contours are shown in Fig. 4. While the images widen somewhat, the conclusions concerning the focusing of the telescope remain unchanged. In particular, it is still possible to obtain a very narrow image of the high-altitude part of the air shower.

#### 4. Concluding remarks

In particular for next-generation Cherenkov telescopes with very large mirrors and fine segmentation of the photon detector, the focusing of the telescope becomes a relevant issue; this is obviously the case once the diameter of the mirror is comparable to the transverse size of an air shower. Optimal imaging of the Cherenkov light from air showers is achieved by focusing the telescope on the relevant portion of an air shower. For a classical Hillas-type analysis, which averages over the longitudinal profile of a shower when determining the width, one should focus on the region of the shower maximum. Advanced image analyses resolving fine details of the image and making use of the sharp



**Figure 4.** Equidistant contour lines of the photon density in Cherenkov images of 100 GeV vertical gamma-ray showers, viewed at a distance of 120 m from the shower axis with a 20 m diameter reflector, focused at different distances (infinity, 15 km). Optical aberrations are not included in the simulation. Shower simulations include the geomagnetic field at La Palma.

core region of the image are better served by focusing on the head of the air shower.

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